Political decision of risk reduction: the role of trust

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Abstract

Surveys concerning environmental and health risks point out the lack of trust of citizens in risk evaluations provided by governments. The aim of this paper is to take into account this potential distrust in political decisions concerning risk reduction. We prove that lack of trust reduces the attractivity of risk reduction measures. When heterogeneity in risk exposure and the possibility of complete risk elimination are introduced, political decisions of risk reduction may differ from the preferred decision of any risk and trust group. Namely, total risk elimination can be adopted, even if all individuals prefer null or partial risk reduction measures.

Keywords: risk, political decisions, Choquet expected utility preferences
JEL code: D7, D81.

1 Introduction

Environmental, as well as health and food risks have taken, in the last few decades, a more and more important place in public debate. This is essentially due to uncertainties about their characteristics that render the determination of optimal decisions concerning these risks' management particularly difficult. The Precautionary principle has been adopted to guide the authorities' actions concerning environmental and health risks in the presence of scientific uncertainties. However, due to its general and imprecise formulation, it allows multiple interpretations that make it difficult to apply as a unique decision criterion (for a discussion on the controversies around the Precautionary Principle, see Godard 2003). Political decisions concerning risk management are generally guided by two types of consideration: scientific knowledge and individual preferences and beliefs. Beliefs are strongly influenced by public

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information, and trust in this information. Several surveys realized in the Member States of the European Union on different topics show that the trust of citizens in their government is far from being complete. For instance, in a Eurobarometer survey in 2004 on "The Attitudes of European citizens towards environment" it appears that only 11% of the respondents trust their national government to inform them about environmental problems. Another Eurobarometer survey in 2005 about radioactive waste confirms this result and even shows a decrease in confidence: it appears that in 2005 19% of the respondents trust their government when it informs them on the treatment of radioactive waste, whereas it was 29% in 2001.

The aim of this paper is to study the impact of the lack of trust in the available information on the political decisions concerning risk reduction. More precisely, we determine the risk reduction level that emerges from a voting process where voters differ both in the confidence they have in the risk evaluation given by the authorities and in their wealth. The obtained choices are discussed with respect to different interpretations of the precautionary principle and more precisely with the more radical interpretation that corresponds to a complete risk elimination.

The trust level is taken into account by a model proposed by Jaffray (1988) and Cohen (1992) under risk and by Eichberger, Kelsey (1999) and Chateauneuf, Eichberger and Grant (2002) under uncertainty. In this model, generalizing expected utility, beliefs are represented by a weighted sum of a probability distribution, corresponding to the initial information, and a capacity, characterizing complete uncertainty. With this choice criterion, a decision is evaluated by a combination of the standard expected utility of the decision and its best and worst consequences. This last element reflects the idea that if one does not believe at all in the available risk evaluation, one will consider oneself to be in a pure state of uncertainty and will take into account only the best and worst possible outcomes, according to his degrees of pessimism-optimism.

The political decision criterion adopted here is that characteristic of a representative democracy modeled by probabilistic voting. The simple direct majority indeed does not seem to be well adapted for the risk decisions considered here for two reasons: (i) it would lead to a complete ignorance of risks affecting a small minority (which is not realistic), (ii) in the case of several groups differing by more that one parameter, equilibrium may not exist. We assume that risk reduction is financed by a tax on wealth at a uniform rate.

Two types of risk are considered: global risks, that equally affect all the voters (such as global warming, GMO authorization and pandemics) and more specific risks for which risk groups can be identified, differing both by the estimated probabilities of the risk occurrencing and by amounts of loss in the case of risk realization (such as risks issued from new drugs, new technologies, chemical plant installations etc.). For global risks, many risk reduction levels are available, going from the absence of any action to a complete risk elimination (if technologically possible). For specific risks, we assume that only three decisions are available: no action (status quo), complete risk elimination (by prohibition of the source of risk) and a given intermediate level of risk reduction.

The main results are the following.

Even if the risk is high, the politically decided level of risk reduction will be low, if the average trust level is low. The impact of wealth on the risk reduction level depends both on the global wealth level in the population and on the wealth distribution between the different distrust level groups. The difference between the risk reduction level emerging from a political process and that preferred by an average individual depends on the individuals' degree of relative risk aversion. Moreover, it appears that in general, the political decision differs from the socially optimal one.

When only three decisions are possible: no action (status quo), partial reduction or complete risk eradication, and when individuals differ not only in their confidence level but also in their objective risk exposure, the optimal choice may differ from the best choices of all the individuals in the population: Condorcet type paradoxes are possible. These situations may occur when credible prohibition is not too costly with respect to partial reduction.

The paper starts with the presentation of the preferences representation model and the determination of the risk reduction level preferred by a given agent. We then study the politically chosen risk reduction level when individuals differ in wealth but are exposed to the same risk. The fourth section is devoted to the political decision when individuals differ in risk exposure.

2 The individually preferred risk reduction level

The aim of this section is to determine the level of risk reduction preferred by a given individual. This would be the implemented risk reduction level if the political decision was taken only with respect to this individual's preferences (or if this agent was a dictator). This analysis provides a benchmark for the general study of the political decision of risk reduction.

2.1 Individual preferences representation

We assume in this section that the population is composed of n individuals with preferences towards wealth characterized by the same increasing and concave utility function u. Each individual faces a risk of loss, resulting from the occurrence of a catastrophic event E. The government provides an estimation of the probability distribution of the individual loss.

We assume that individuals do not completely trust the risk characteristics provided by the government. At least two reasons can explain this distrust:

- the risk is new and experts disagree on the estimation of the catastrophe probability that renders the "official" estimation, often based on an average value, not reliable.

- citizens have more general reasons not to trust the government.

This lack of confidence in probability estimations is well taken into account by a model in the Choquet Expected utility class proposed by Chateauneuf, Eichberger, Grant (2002).

In this model, beliefs are characterized, not by a probability distribution, but by a neo-additive capacity, defined in the following way:

Definition 1 *Let Ω be a state space and $\mathcal{A}$ a σ-algebra of subsets of Ω. μ^0 and μ^1 are the capacities defined as follows:*

- $\mu^0(\Omega) = 1$ *and* $\mu^0(A) = 0$ *for all* $A \in \mathcal{A}$*, with* $A \neq \Omega$*;*
- $\mu^1(\varnothing) = 0$ *and* $\mu^1(A) = 1$ *for all* $A \in \mathcal{A}$*, with* $A \neq \varnothing$*.*

For a given finitely additive probability distribution P on $(\Omega, \mathcal{A})$, a neo-additive capacity ν is defined as:
$\nu(A) = (1 - \varepsilon - \gamma)P(A) + \varepsilon\mu^0(A) + \gamma\mu^1(A)$ *with* $\varepsilon \geq 0, \gamma \geq 0$ *and* $\varepsilon + \gamma \leq 1$*.*

A neo-additive capacity is then a convex combination of a probability measure and two capacities, reflecting complete ignorance.

$\varepsilon + \gamma$ measures to which extent beliefs are far from some estimated probabilities.

In our context, the individual loss is a random variable X, taking its value in $\{x_0, x_1, ..., x_l\}$, with $0 = x_0 < x_1 < ... < x_l = b$, where b is the maximal possible loss. The government gives the distribution P of X, i.e. the value of $P(X = x_i)$ for every i. The catastrophic event is $E = \{X > 0\}$, its probability estimated by the government is $p = P(E)$, and we set $V = E(X/X > 0)$

The neo-capacity ν here associated with the official distribution P of X gives:
$\nu(E) = (1 - \varepsilon - \gamma)p + \gamma$ and $\nu(\overline{E}) = (1 - \varepsilon - \gamma)(1 - p) + \gamma$.

The preferences representation *à la Choquet* of an individual with initial wealth y and with beliefs represented by the neo-capacity ν can be written as:

$$\begin{aligned} W_\nu(y) &= (1 - \varepsilon - \gamma)E_P(u(y) - X(\omega)) + \varepsilon \inf_{\omega \in \Omega} (u(y) - X(\omega)) + \gamma \sup_{\omega \in \Omega} (u(y) - X(\omega)) \\ &= (1 - \varepsilon - \gamma)(u(y) - pE(X/X > 0)) + \varepsilon(u(y) - b) + \gamma(u(y) - 0) \\ &= u(y) - \varepsilon b - p(1 - \varepsilon - \gamma)V \end{aligned}$$

$1 - \varepsilon - \gamma$ measures the trust (or confidence) level in the government probability estimation.

Note that, for $\varepsilon = \gamma = 0$, the beliefs coincide with the official estimation and the standard expected utility evaluation is obtained. When distrust appears, $(\varepsilon, \gamma > 0)$, ε and γ reflect respectively the individual's pessimism and optimism. Indeed, the maximal possible loss appears explicitly in the agent's utility evaluation and its impact increases with an increase in ε., the minimal possible loss appears too with weight γ but is implicit because its value here is 0.

In the following, we set $\delta = \varepsilon + \gamma$ the distrust level and thus the previous preferences representation becomes:

$$W_{\varepsilon,\delta}(y) = u(y) - \varepsilon b - p(1 - \delta)V \text{ where } 0 \leq \varepsilon \leq \delta \leq 1.$$

We see that the utility of the agent depends on the pessimism ε and on the distrust level δ. The optimism intervenes only through the distrust δ.

2.2 The risk reduction level

A possibility of risk reduction is now introduced for the whole population. This risk reduction can take two forms, the first one corresponding to self-protection, and the other one to self insurance in the sense of Ehlrich, Becker (1972):

- a reduction of the *probability* of the catastrophic event which decreases from p to $(1-\lambda)p$, with $0 \leq \lambda \leq 1$;

- a reduction of the *average loss* in the case of occurrence of E, which decreases from V to $(1-\lambda)V$, with $0 \leq \lambda \leq 1$.

The difference between the first two types of measure well appears in the following example concerning the threat of an influenza pandemic. The WHO elaborates prevention plans with two objectives. The first objective is to avert a pandemic and to control the outbreak in humans. Measures then consist in immediate culling of infected and exposed birds, quarantine and disinfection of farms, control of animal movements etc. The second objective is to mitigate the impact of the pandemic (once it has been declared). The corresponding measures include vaccine development and antiviral drug production and storage.

Note that the two types of risk reduction are equivalent in our model because the utility function is additively separable.

We assume that the reduction of pV to $(1-\lambda)pV$ with $0 \leq \lambda \leq 1$ costs $T(\lambda)$.This cost is financed by a uniform tax rate $t(\lambda, Y) = \frac{T(\lambda)}{Y}$ where Y is the total wealth in the economy. $\lambda \mapsto T(\lambda)$ is assumed to be an increasing and convex function with $T(0) = 0$. Thus if $\lambda = 0$, the risk remains at its initial level, and if $\lambda = 1$ the risk is completely eliminated according to the government. However, if the agent is pessimistic (i.e. $\varepsilon > 0$), he will believe that the maximal loss is still possible even if $\lambda = 1$ (see the term $-\varepsilon b$ in Equation (1)).

For an agent with distrust level δ, degree of pessimism ε, and income y, the evaluation of a risk reduction level λ gives:

$$W_{\varepsilon,\delta}(y, \lambda) = u\left(y(1 - t(\lambda, Y))\right) - \varepsilon b - (1-\lambda)p(1-\delta)V \tag{1}$$

The optimal risk reduction level λ^* for this agent is then the solution of the following maximization problem:

$$\max_{\lambda} \; u\left(y(1 - t(\lambda, Y))\right) - \varepsilon b - (1-\lambda)p(1-\delta)V \tag{2}$$

The first order condition for an internal solution is:

$$\frac{yT'(\lambda)}{Y} u'\left(y(1 - t(\lambda, Y))\right) = p\left(1-\delta\right)V \tag{3}$$

The second order condition is satisfied for all $\lambda \in [0, 1]$ due to the concavity of u and to the convexity of T.

The optimal risk level equalizes the marginal benefit of risk reduction $p\left(1-\delta\right)V$ with its marginal cost in terms of utility $\frac{yT'(\lambda)}{Y} u'\left(y(1 - t(\lambda, Y))\right)$.

Note that the optimal risk reduction level $\lambda^* = \lambda^*(\delta, y)$ does not depend specifically on ε, but only on the global distrust level δ.

In the special case of logarithmic function: $u(x) = \ln x$, the first order condition becomes:

$$\frac{T'(\lambda)}{(Y - T(\lambda))} = p\,(1 - \delta)\,V \tag{4}$$

Let us now consider the conditions for corner solutions.

Maintaining risk at its actual level is optimal if and only if the solution of the optimization problem (2) is $\lambda^* = 0$ which is equivalent to the following condition:

$$\frac{yT'(0)}{Y} u'\,(y) \geq p\,(1 - \delta)\,V$$

Thus, preserving the status quo is preferred if the marginal cost of risk reduction is higher than its marginal benefit "from the first euro". This choice will be less likely to occur for severe risks, that is when pV is high.

On the other hand, the maximal risk reduction is adopted if and only if the solution of the maximization program (2) is $\lambda^* = 1$ which is equivalent to the following condition:

$$\frac{yT'(1)}{Y} u'\,(y(1 - t(1, Y))) \leq p\,(1 - \delta)\,V$$

The maximal risk reduction level is preferred if the marginal benefit of risk reduction is higher than its marginal cost until the last risk fraction. This situation is possible only if $T(1)$ and $T'(1)$ are finite, which is not always the case, namely for natural disasters.

3 The political decision of risk reduction

We consider now a population composed of n individuals differing by their trust in the estimation of the risk announced by the government, by their pessimism, their wealth and their political weight. We aim to determine the level of risk reduction which will be implemented by the government.

3.1 The model of political decision

To simplify, we assume that there are k groups of homogenous individuals of size n_i, with $i = 1, .., k$; their parameters of distrust and wealth are respectively δ_i and y_i with $i = 1, .., k$. Pessimism and optimism levels are ε_i and γ_i respectively, with $\varepsilon_i + \gamma_i = \delta_i$. However, within a group, the agents can have preferences on other matters than the catastrophic risk. We model the political decision with probabilistic voting (see Persson, Tabellini (2000), Coughlin, Mueller, Murell (1990), Lindbeck, Weibull (1987, 1993)). Two parties A and B compete in elections. Each party announces a policy, and it is assumed that the policy announced by the winning party will be implemented.

More precisely, the agent j of group i votes for party A iff

$$W_{\varepsilon_i,\delta_i}(y_i, \lambda_A) + b_{i,j} > W_{\varepsilon_i,\delta_i}(y_i, \lambda_B)$$

where λ_A and λ_B are the policies announced respectively by parties A and B. The welfare of any agent of group i is $W_{\varepsilon_i,\delta_i}(y_i,\lambda)$ if the policy λ is applied. The random variable $b_{i,j}$ measures the bias (positive or negative) of elector j in favor of party A, independently of policy λ. Within group i, the random variables $b_{i,j}$ have the same law as a random variable b_i, which is assumed to have a continuous law.

The mathematical expectation of the number of votes for A is

$$EW^A(\lambda_A,\lambda_B) = \sum_i \sum_j P\left(W_{\varepsilon_i,\delta_i}(y_i,\lambda_A) + b_{i,j} > W_{\varepsilon_i,\delta_i}(y_i,\lambda_B)\right)$$

As all the $b_{i,j}$ have the same law as b_i, we have

$$EW^A(\lambda_A,\lambda_B) = \sum_i n_i P\left(W_{\varepsilon_i,\delta_i}(y_i,\lambda_A) + b_i > W_{\varepsilon_i,\delta_i}(y_i,\lambda_B)\right)$$

and the mathematical expectation of the number of votes for B is

$$EW^B(\lambda_A,\lambda_B) = \sum_i n_i P\left(W_{\varepsilon_i,\delta_i}(y_i,\lambda_A) + b_i < W_{\varepsilon_i,\delta_i}(y_i,\lambda_B)\right)$$

Let F_i denote the cumulative distribution function of b_i and f_i be its density. We obtain:

$$EW^B(\lambda_A,\lambda_B) = \sum_i n_i F_i\left(W_{\varepsilon_i,\delta_i}(y_i,\lambda_B) - W_{\varepsilon_i,\delta_i}(y_i,\lambda_A)\right)$$

and

$$EW^A(\lambda_A,\lambda_B) = n - EW^B(\lambda_A,\lambda_B)$$

where $n = \sum_{i=1}^k n_i$ is the total number of agents.
Party B chooses λ_B to maximize $EW^B(\lambda_A,\lambda_B)$ (for λ_A given). Idem for Party A. Thus the first order conditions are:

$$\begin{aligned} 0 &= \sum_i n_i \frac{\partial W_{\varepsilon_i,\delta_i}(y_i,\lambda_B)}{\partial \lambda_B} f_i\left(W_{\varepsilon_i,\delta_i}(y_i,\lambda_B) - W_{\varepsilon_i,\delta_i}(y_i,\lambda_A)\right) \\ 0 &= \sum_i n_i \frac{\partial W_{\varepsilon_i,\delta_i}(y_i,\lambda_A)}{\partial \lambda_A} f_i\left(W_{\varepsilon_i,\delta_i}(y_i,\lambda_B) - W_{\varepsilon_i,\delta_i}(y_i,\lambda_A)\right) \end{aligned}$$

The two parties face the same problem. Thus at the Nash equilibrium, with simultaneous announcement of the policies, we have $\lambda_A = \lambda_B$, i.e.:

$$0 = \sum_i n_i \frac{\partial W_{\varepsilon_i,\delta_i}(y_i,\lambda_B)}{\partial \lambda_B} f_i(0)$$

which is the FOC corresponding to the maximization of $\sum_i n_i \alpha_i W_{\varepsilon_i,\delta_i}(y_i,\lambda_B)$ for $\alpha_i = f_i(0)$.

We see that the political equilibrium implements the maximum of a sort of social welfare function, where each elector of group i is considered to have a weight $\alpha_i = f_i(0)$.

$f_i(0)$ is the density function of b_i at the equilibrium. A high $f_i(0)$ means that the electors of group i will change their vote more easily if the policy proposed is modified. The political equilibrium then gives a greater weight to the individuals who are more prompt to change their vote.

3.2 Adopted risk reduction level

With the previous political decision modelization, the adopted risk reduction level λ^{**} will be the solution of the following optimization program:

$$\max_{\lambda} D(\lambda) = \sum_{i=1}^{k} \alpha_i n_i \left[u\left(y_i (1 - t(\lambda, Y)) \right) - \varepsilon_i b - (1-\lambda) p (1-\delta_i) V \right]$$

where $t(\lambda, Y)$ is the uniform tax rate which finances a risk reduction corresponding to λ, given by $t(\lambda, Y) = \frac{T(\lambda)}{Y}$ with $Y = \sum_{i=1}^{k} n_i y_i$.

The agents choose to vote for A or B. At the equilibrium, parties A and B choose the same λ to maximize the electoral support function $D(\lambda)$, taking the parameters α_i, δ_i, ε_i, n_i, y_i, Y, V, p, b as fixed.

The first order condition for an internal solution is:

$$D'(\lambda) = 0 \tag{5}$$

where

$$\begin{aligned} D'(\lambda) &= \sum_{i=1}^{k} \alpha_i n_i \left[-T'(\lambda) \frac{y_i}{Y} u'\left(y_i \left(1 - \frac{T(\lambda)}{Y} \right) \right) + p(1-\delta_i)V \right] \\ &= -\frac{T'(\lambda)}{Y} \left[\sum_{i=1}^{k} \alpha_i n_i y_i u'\left(y_i \left(1 - \frac{T(\lambda)}{Y} \right) \right) \right] + p(1-\overline{\delta})V \sum_{i=1}^{k} \alpha_i n_i \end{aligned} \tag{6}$$

with $\overline{\delta} = \frac{\sum_{i=1}^{k} \alpha_i n_i \delta_i}{\sum_{i=1}^{k} \alpha_i n_i}$.

Morover, setting $\overline{u}'(\lambda) = \frac{1}{\sum\limits_{i=1}^{k} \alpha_i n_i} \sum\limits_{i=1}^{k} \alpha_i n_i \frac{y_i}{Y} u'\left(y_i (1 - \frac{1}{Y} T(\lambda)) \right)$, the first order condition becomes:

$$T'(\lambda) \overline{u}'(\lambda) = pV(1-\overline{\delta}) \tag{7}$$

The second order condition is satisfied for any $\lambda \in [0,1]$ because of the concavity of u and the convexity of T. As in (3), we obtain in (7) the equality of marginal utility and marginal cost of risk reduction. Note that, as in section 2.2, the adopted risk reduction level depends here only on $\overline{\delta}$, and not specifically on the pessimism $\overline{\varepsilon}$.

It appears that preferences and trust levels of the different groups intervene in the government decision criterion via an "average" distrust level $\overline{\delta}$ and an "average" marginal cost (in terms of utility) $\overline{u}'(\lambda)$ which both depend not only on the size of each group, but also on their respective political weights.

In the particular case of a logarithmic utility function, $u(x) = \ln x$, the first order condition becomes:

$$\frac{T'(\lambda)}{(Y - T(\lambda))} = p\left(1 - \overline{\delta}\right) V \tag{8}$$

This case has two specific features:

- only the total wealth in the economy influences the adopted level of risk reduction, the distribution of wealth between the groups of individuals plays no role.

- the political weight of a group has an influence on the adopted risk reduction level only via the average trust level in the population.

3.3 Some comparative static results

3.3.1 Impact of risk and trust level

In this section, we study the impact of an increase in the estimated probability of risk realization and the impact of an increase in the average distrust level on the politically decided investment in risk reduction.

Proposition 1 $\frac{d\lambda^{**}}{dp} > 0$ *and* $\frac{d\lambda^{**}}{d\overline{\delta}} < 0$.

Proof

From the first order condition (5), we have $\frac{d\lambda^{**}}{dp} = -\frac{1}{D''_{\lambda\lambda}}\frac{\partial D'_{\lambda}}{\partial p}$ and $\frac{d\lambda^{**}}{d\overline{\delta}} = -\frac{1}{D''_{\lambda\lambda}}\frac{\partial D'_{\lambda}}{\partial \overline{\delta}}$ where $D''_{\lambda\lambda} < 0$ from the second order condition of the optimization program.

Moreover, $\frac{\partial D'_{\lambda}}{\partial p} = V\sum_{i=1}^{k}\alpha_i n_i(1-\delta_i) > 0$ and $\frac{\partial D'_{\lambda}}{\partial \overline{\delta}} = -pV\sum_{i=1}^{k}\alpha_i n_i < 0$.■

- The first result implies that an increase in announced loss probability, ceteris paribus, leads to a higher investment in risk reduction. That corresponds to the standard results on optimal prevention, obtained by Elrich, Becker (1972).

- From the second result, the optimal level of risk reduction decreases when average trust in government announcements deteriorates, i. e. when $\overline{\delta}$ increases. This result is specific to the adopted decision model and well emphasizes the particular role played by the trust level in the individual belief formation and in individual preferences. Indeed, the impacts of p and $\overline{\delta}$ on λ^{**} are opposite: if the government announces a higher probability of catastrophe, then the investment in risk reduction increases, whereas a decrease in the trust level decreases the percieved efficiency of risk reduction (only the expected loss being reduced and not the maximal one), and thus decreases investment in risk reduction. Note that increases of optimism and of pessimism have the same impact on λ^{**} because they both induce a lower trust in risk reduction. One can note in addition that an increase in $\overline{\delta}$ can come here not only from a decrease in trust due to some government action, but also from an increase in the political weight of the group with the lower trust level (corresponding here to the higher δ_i).

3.3.2 Impact of wealth and political weight

Wealth can influence λ^{**} by two channels: via the global wealth in the economy and via the distribution of this wealth. We denote by $\beta_i = \frac{y_i}{Y}$ the proportion of the total wealth belonging to an individual of group i , for $i = 1,..., k$ and determine in the following proposition the impact of two types of wealth modifications: a proportional increase of all incomes (leaving β_i constant for any i), and a simple redistribution between two groups i and j (leaving Y constant).

Proposition 2 *(i)* $\left.\dfrac{\partial \lambda^{**}}{\partial Y}\right|_{d\beta=0} > 0.$

(ii) for a CRRA utility function ($u(x) = \ln x$ for $R = 1$, and $u(x) = \dfrac{x^{1-R}}{1-R}$ for $R \neq 1$),

- if $R = 1$ then $\left.\dfrac{\partial \lambda^{**}}{\partial y_i}\right|_{dY=0} = 0;$

- if $R \neq 1$, then $\left.\dfrac{\partial \lambda^{**}}{\partial y_i}\right|_{dY=0,\ dy_l=0,\ l\neq i,\ l\neq j} > 0 \Leftrightarrow (R-1)\left(\dfrac{\alpha_i}{y_i^R} - \dfrac{\alpha_j}{y_j^R}\right) > 0.$

λ^{**} increases with the total wealth in the economy, the impact of a modification in the wealth distribution depends on the relative risk aversion.

Proof

From (5) and (6), $\left.\dfrac{\partial \lambda^{**}}{\partial Y}\right|_{d\beta=0} = -\frac{1}{D''_{\lambda\lambda}} \left.\dfrac{\partial D'_\lambda}{\partial Y}\right|_{d\beta=0}$ and $\left.\dfrac{\partial \lambda^{**}}{\partial y_i}\right|_{d\beta=0} = -\frac{1}{YD''_{\lambda\lambda}} \left.\dfrac{\partial D'_\lambda}{\partial \beta_i}\right|_{dY=0}$ where $D''_{\lambda\lambda} < 0$ from the second order condition of the optimization program.

(i) From (6) and $\beta_i = \frac{y_i}{Y}$, we have $D'_\lambda = \sum\limits_{i=1}^{k} \alpha_i n_i \left[-T'(\lambda)\beta_i u'\left(\beta_i\left(Y - T(\lambda)\right)\right) + p(1-\delta_i)V\right],$ so that:

$\left.\dfrac{\partial D'_\lambda}{\partial Y}\right|_{d\beta=0} = -T'(\lambda)\sum_{i=1}^{k} \alpha_i n_i \beta_i^2 u"\left(\beta_i\left[Y - T(\lambda)\right]\right) > 0$ and thus $\left.\dfrac{\partial \lambda^{**}}{\partial Y}\right|_{d\beta=0} > 0$

(ii) if $R = 1$, $\left.\dfrac{\partial \lambda^{**}}{\partial y_i}\right|_{dY=0} = 0$ because, from (8), λ^{**} depends only on Y.

if $R \neq 1$ $\left.\dfrac{\partial D'_\lambda}{\partial \beta_i}\right|_{dY=0} = -T'(\lambda)\frac{(1-R)}{(Y-T(\lambda))^R}\left(\dfrac{\alpha_i n_i}{\beta_i^R} - \dfrac{\alpha_j n_j}{\beta_j^R}\dfrac{n_i}{n_j}\right)$ which has the sign of $(R-1)\left(\dfrac{\alpha_i}{y_i^R} - \dfrac{\alpha_j}{y_j^R}\right)$. ■

Consequently, it appears from (i) that risk reduction is as a normal good: investment in it increases with global wealth.

The impact of a redistribution depends on the relative risk aversion R. When the utility function is logarithmic, the risk reduction level is neutral concerning any redistribution of wealth between individuals in the population: only the total wealth matters. Thus if a fiscal reform modifies the distribution of wealth, with global wealth fixed, it will not change the risk reduction level.

On the other hand, when the utility function is CRRA with $R \neq 1$, the previous neutrality property no longer holds: a change in wealth distribution influences λ^{**}

even if the total wealth remains constant. This is due to the fact that when $R \neq 1$ the marginal cost of risk reduction depends not only on the total wealth in the economy, but also on wealth distribution. Then, the variation of λ^{**} in the case of an increase in y_i, Y being constant, will result from two effects: group i becomes richer and thus, for $R > 1$, prefers more risk reduction (the marginal cost for risk reduction becomes lower for its members), whereas group j becomes less rich and thus prefers less risk reduction (the marginal cost for risk reduction becomes higher for its members)[1]. Note that, for $R = 1$, the two effects compensate perfectly.

The following proposition proves an intuitive result: an increase in the political weight of the individuals of a given group increases the politically decided level of risk reduction if and only if the individuals in this group are in favour of a high risk reduction level.

Proposition 3 $\frac{d\lambda^{**}}{d\alpha_i} > 0 \Leftrightarrow \lambda^*(\delta_i, y_i) > \lambda^{**}$.

Proof

From (5) and (6), $\dfrac{d\lambda^{**}}{d\alpha_i} = -\frac{1}{D''_{\lambda\lambda}} \frac{\partial D'_\lambda}{\partial \alpha_i}$ which has the sign of $\frac{\partial D'_\lambda}{\partial \alpha_i}$.

$\frac{\partial D'_\lambda}{\partial \alpha_i} = n_i \left(\frac{-y_i T'(\lambda^{**})}{Y} u'(y_i[1 - \frac{T(\lambda^{**})}{Y}]) + (1-\delta_i)pV \right) = n_i G'_i(\lambda^{**})$

where $G_i(\lambda)$ is defined as $G_i(\lambda) = W_{\varepsilon_i,\delta_i}(y_i, \lambda)$ (see Equation (1)).

$G''_i(\lambda) < 0$, i.e. G'_i is decreasing, and $G'_i(\lambda^*(\delta_i, y_i)) = 0$ thus $G'_i(\lambda^{**}) > 0 \Leftrightarrow \lambda^*(\delta_i, y_i) > \lambda^{**}$. ■

3.3.3 Comparison of λ^{**} with the individually and socially optimal risk reduction levels

In the following we compare the politically decided risk reduction level λ^{**} with the individually prefered risk reduction levels, and particularly the level preferred by an "average" individual, i.e. of average wealth $\bar{y} = \frac{\sum_i \alpha_i n_i y_i}{\sum_i \alpha_i n_i}$ and average trust level $\bar{\delta}$.

Proposition 4 *(i)* $\lambda^{**} \in [\min_i \lambda^*(\delta_i, y_i),\ \max_i \lambda^*(\delta_i, y_i)]$;
(ii) for a CRRA utility function u:
- *if* $R = 1$*, then* $\lambda^{**} = \lambda^*(\bar{\delta}, \bar{y})$ *and* $\lambda^{**} < \lambda^*(\delta_i, y_i) \Leftrightarrow \delta_i < \bar{\delta}$
- *if* $R \neq 1$*, then* $\lambda^{**} < \lambda^*(\delta_i, y_i) \Leftrightarrow \frac{u(y_i)}{\bar{u}(y_1,...,y_k)} < \frac{1-\delta_i}{1-\bar{\delta}}$

  where $\bar{u}(y_1, ..., y_k) = \frac{\sum_{j=1}^k \alpha_j n_j u(y_j)}{\sum_{j=1}^k \alpha_j n_j}$.

  Moreover:

  - if $R > 1$ *then* $\lambda^*(\bar{\delta}, \bar{y}) > \lambda^{**}$

  - if $0 < R < 1$ *then* $\lambda^*(\bar{\delta}, \bar{y}) < \lambda^{**}$

(iii) for any utility function u, if $y_i = y\ \forall i$ *then* $\lambda^{**} = \lambda^*(\bar{\delta}, y)$.

Proof

See Appendix

[1]Note that the opposite holds for $R < 1$.

Concerning the comparison of λ^{**} with the individually optimal risk reduction levels, it appears that λ^{**} lies in the interval between the minimal and the maximal individually prefered risk reduction level, which means that it corresponds to a compromise between the individually prefered risk reduction levels. We will prove in section 4 that this is not always the case when the risk exposure is differentiated.

For a given individual i, the gap between λ^{**} and $\lambda^*(\delta_i, y_i)$ depends both on individual wealth and trust level. For instance, for $0 < R < 1$, an individual will be in favour of more risk reduction than the politically decided if he is more confident and less wealthy than the average individual in the population.

Concerning the comparison of λ^{**} with $\lambda^*(\overline{\delta}, \overline{y})$, when wealth is equally distributed between individuals or when the utility function is logarithmic ($R = 1$), the risk level adopted by the government does not differ from the one preferred by an individual with an average level of trust. It is however important to note that this average trust level depends not only on the respective sizes of the population groups, given by n_i, but also on their respective political weights, given by α_i. Thus, the trust level of group i will influence the decision more if this group is big and if its political weight is important.

For more general preferences, the comparison of λ^{**} and $\lambda^*(\overline{\delta}, \overline{y})$ depends on the degree of concavity of the utility function (measured by R). If R is higher than 1, the marginal utility of wealth is strongly decreasing, and the willingness to pay for risk reduction is proportionally much lower for poor people than for rich ones. That is why the political decision corresponds to less risk reduction than in the case where an "average" individual is considered.

In the following, we compare the risk reduction level λ^{**} resulting from a political process with the utilitarian risk reduction level λ^{opt} which maximizes the social welfare function $D(\lambda)$ corresponding to $\alpha_i = \alpha_j \ \forall i, j$, i. e. when the influence of each group corresponds to its demographic weight. It means that we compare a positive result λ^{**} with a normative one λ^{opt}.

Remark 1 *Let λ^{opt} be the utilitarian optimal risk reduction level (obtained with identical political weight α for every agent). In general, $\lambda^{opt} \neq \lambda^{**}$. However, $\lambda^{opt} \in [\min_i \lambda^*(\delta_i, y_i),\ \max_i \lambda^*(\delta_i, y_i)]$.*

More precisely, if $\alpha_i > \alpha$ for i such that $\lambda^*(\delta_i, y_i) > \lambda^{opt}$ and if $\alpha_i < \alpha$ for i such that $\lambda^*(\delta_i, y_i) < \lambda^{opt}$ then $\lambda^{**} > \lambda^{opt}$.

It appears that in general, the positive and the normative risk reduction levels are different. In particular, the positive level will be higher than the normative one if the individuals preferring higher risk reduction are the more politically influential ones.

4 Political decision with differentiated risk exposure

In this section, we consider risks deriving from a new product or a new technology, for which different risk groups may be identified: we assume more precisely that some individuals, by their location, or by their specific characteristics, are more exposed than others to risk. Another specificity of these risks is the possibility, for the public authorities, to eliminate them completely by forbidding the trade of the products or the use of the technology. In this case, since it is easier to verify that a product is forbidden than to evaluate the risk it generates when it is allowed, the government will be credible if it announces a total ban. Note however that forbidding a product or a technology has a cost (of opportunity for example), since nobody will be able to use the product. This radical solution to the risk exposure problem can be opposed to two other solutions: risk reduction of level λ ($\lambda \in]0;1[$), or to totaly acceptance of the risk.

Three scenarios are then possible: authorization, reduction of risk, and prohibition.

We assume here that the agents differ by their risk p_i and their degrees of pessimism and optimism ε_i and $\gamma_i = \delta_i - \varepsilon_i$, where δ_i is the degree of distrust. It means that, given the individual characteristics (age, profession, localization etc.), each agent i has an individual probability p_i of realization of the risk. The associated utility loss is V_i, and p_i is the probability announced by the government for an agent of type $\dot{i}$. The trust of agent i in the evaluation of the government is $1 - \delta_i$. In the three cases, we can evaluate the utility of an agent, which is a function of her income y, of her risk p_i, of her degree of pessimism ε_i and of her degree of trust $1 - \delta_i$.

Authorization

$$W^i_{author}(y) = u(y) - \varepsilon_i b - V_i p_i (1 - \delta_i)$$

Reduction of risk by a factor λ

$$W^i_{red}(y) = u\left(y(1-t)\right) - \varepsilon_i b - (1-\lambda) V_i p_i (1 - \delta_i)$$

Prohibition

$$W^i_{prohib}(y) = u\left(y(1-\theta)\right)$$

where θy represents the cost of prohibition.

To simplify, we assume that if the reduction of risk is chosen, this will be of a factor $\lambda \in]0;1[$, λ given. To avoid the obvious case of prohibition preferred to reduction for every ε_i, δ_i, p_i, we assume that $\theta > t > 0$.

In the following, we set $Q_i = p_i V_i (1 - \delta_i)$.

4.1 Individual preference

We can examine now which scenario is preferred by an agent of type i: authorization, reduction of risk or prohibition.

We set:

$E_1 = \left\{(\varepsilon_i, Q_i); W^i_{author}(y) = \max\left(W^i_{author}(y); W^i_{red}(y); W^i_{prohib}(y)\right)\right\}$

$E_2 = \left\{(\varepsilon_i, Q_i); W^i_{red}(y) = \max\left(W^i_{author}(y); W^i_{red}(y); W^i_{prohib}(y)\right)\right\}$

$E_3 = \left\{(\varepsilon_i, Q_i); W^i_{prohib}(y) = \max\left(W^i_{author}(y); W^i_{red}(y); W^i_{prohib}(y)\right)\right\}$

An agent with individual characteristics (ε_i, Q_i) prefers the authorization iff $(\varepsilon_i, Q_i) \in E_1$. She prefers reduction of risk by a factor λ if $(\varepsilon_i, Q_i) \in E_2$, and she prefers the prohibition if $(\varepsilon_i, Q_i) \in E_3$.

Let us denote by $z^*(\varepsilon_i, Q_i)$ the individual preference of an individual of characteristics (ε_i, Q_i), where we set $z^*(\varepsilon_i, Q_i) = j$ if $(\varepsilon_i, Q_i) \in E_j$, with $j \in \{1, 2, 3\}$.

Thus, $z^*(\varepsilon_i, Q_i) = 1$ means that individual i prefers authorization, $z^*(\varepsilon_i, Q_i) = 2$ means that individual i prefers risk reduction and $z^*(\varepsilon_i, Q_i) = 3$ means that individual i prefers prohibition.

The following proposition gives some results about the impact of ε_i, γ_i and p_i on the individual preferences $z^*(\varepsilon_i, Q_i)$; recall that $Q_i = p_i V_i (1 - \varepsilon_i - \gamma_i)$

Proposition 5 *$z^*(\varepsilon_i, Q_i)$ is an increasing function with respect to p_i and V_i; a decreasing function of γ_i, and an increasing function of ε_i (Q_i being fixed).*

Proof
See Appendix

Proposition 5 means that an increase in the estimated risk p_i or in the pessimism level ε_i leads to a more cautious preferred decision concerning risk. Note that an increase in ε_i may modify the decision from authorization to prohibition or from reduction to prohibition, but never from authorisation to reduction.

Let us compare the results about the influence of the pessimism ε given in Proposition 1 and in Proposition 5. In Proposition 1, an increase of pessimism induces less risk reduction (since $\delta = \gamma + \varepsilon$). In Proposition 5, an increase of pessimism can lead to prohibition, since here prohibition is a trustable way to eliminate risk.

We can distinguish two cases for the shape of the sets E_j according to the values of the costs (θ and t) and benefits (λ) of risk reduction levels.

- Case 1: if $\frac{u(y)-u(y(1-\theta))}{u(y)-u(y(1-t))} \leq \frac{1}{\lambda}$, the reduction of risk is never preferred, since prohibition is hardly more expensive than reduction of risk (see Figure 1).

- Case 2: if $\frac{1}{\lambda} \leq \frac{u(y)-u(y(1-\theta))}{u(y)-u(y(1-t))}$, the reduction of risk can be preferred since prohibition is quite more expensive than reduction of risk.(see Figure 2).

4.2 The political choice

In this section, we consider the political choice concerning risk when the three previous possibilities can be chosen by the government. The analysis draws several scenarios which depend essentially on the relative cost of prohibition with respect to the reduction of risk, and on the impact of the reduction of risk on the maximum loss. The main results are summed up at the end of the section.

We keep the model of the political choice by probabilistic voting. The population is constituted of k groups of homogenous individuals. Group i is composed of agents

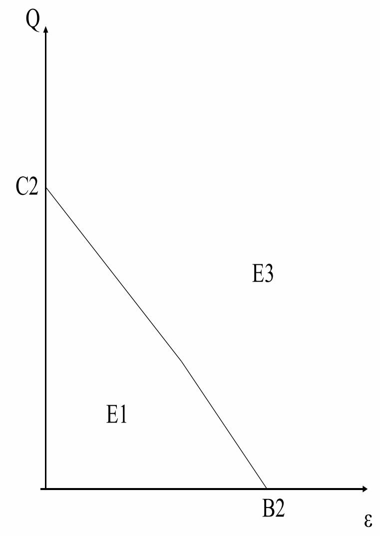

Figure 1: case 1

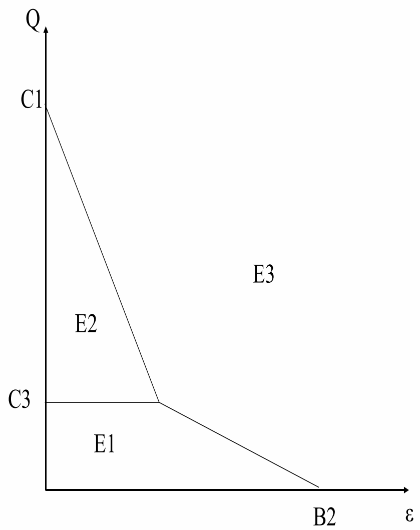

Figure 2: case 2

having the same characteristics (ε_i, Q_i). The chosen policy maximizes the political decision function:

$$D = \sum_{i=1}^{k} n_i \alpha_i W(\varepsilon_i, Q_i)$$

where α_i is the political weight of group i (as in section 3). Three policies are possible: authorization, reduction of risk by a given factor λ, and prohibition.

We denote by z^{**} the political decision. We have $z^{**} = 1$ if authorization is decided, $z^{**} = 2$ if reduction is decided, and $z^{**} = 3$ if it is prohibition.

Proposition 6 *(i)* $z^{**} = z^*(\overline{\varepsilon}, \overline{Q})$*, i.e. the political decision is that of an average individual*

(ii) $z^*(\varepsilon_i, Q_i) = z^*(\varepsilon_j, Q_j)$*, for any* $i, j = 1, ..., k \Rightarrow z^{**} = z^*(\varepsilon_i, Q_i)$ *because* E_j *is a convex set for any* $j = 1, 2, 3$*. If the individuals have the same preference, the political decision conforms to their wish.*

(iii) In case 1, $z^{**} \in conv\{z^*(\varepsilon_i, Q_i), i = 1, ..., k\}$*;*

(iv) In case 2, $z^{**} \notin conv\{z^*(\varepsilon_i, Q_i), i = 1, ..., k\}$ *is possible because* $E_1 \cup E_2$ *and* $E_2 \cup E_3$ *are non-convex sets.*

Proof : See Appendix □

The political decision $z^{**} = z^*(\overline{\varepsilon}, \overline{Q})$ is that of an average individual even if nobody has his preferences. However, in case 1, the preference of the average agent is a compromise between the preferences of the different agents, i. e. $z^{**} \in conv\{z^*(\varepsilon_i, Q_i), i = 1, ..., k\}$. For instance, if nobody prefers prohibition, it will not be adopted.

At the opposite, in case 2, the preference of an average agent can be extreme, i.e. $z^{**} \notin conv\{z^*(\varepsilon_i, Q_i), i = 1, ..., k\}$ is possible. For instance, even if nobody prefers prohibition, it can be adopted.

Thus, an extreme interpretation of the precautionary principle rejecting any risk may come from a political process, even when every elector accepts a partial or total risk. It can be due to a diversity of risk exposures, combined with a difference of trust levels.

The following remark studies the impact of political weights on the political decision.

Remark 2 *When* α_i *increases, with* α_j *fixed for all* $j \neq i$*, then* $(\overline{\varepsilon}, \overline{Q})$ *describes a segment of line and the political decision* $z^{**} = z^*(\overline{\varepsilon}, \overline{Q})$ *moves consequently. In case 1,* z^{**} *variates monotonously. This is no longer true in case 2. For example, we can have* $z^{**} = 2$ *for* α_1 *low,* $z^{**} = 3$ *for* α_1 *medium and* $z^{**} = 1$ *for* α_1 *high.*

Finally, in a last remark, we compare the utilitarian socially optimal decision z^{opt} with the political decision z^{**}. We note that z^{opt} may be an extreme decision, which was not possible in the framework of section 3 (see Remark 1).

Remark 3 *Let* z^{opt} *be the utilitarian socially optimal decision, i. e. obtained maximizing the social welfare function* $D(\lambda)$ *corresponding to* $\alpha_1 = \alpha_2 = ... = \alpha_k$*. In general,* $z^{**} \neq z^{opt}$*. Moreover, in case 2 we can have* $z^{opt} \notin conv\{z^*(\varepsilon_i, Q_i), i = 1, ..., k\}$ *and then the socially optimal decision can be extreme.*

5 Conclusion

This paper is an attempt to introduce non probabilized uncertainty in a political economy model. More precisely, an estimated probability of a catastrophic event is given to the agents who may more or less trust it.

Increases in estimated probability and in distrust do not have the same impact on risk reduction decisions: if an increase in the estimated probability always leads to more risk reduction, an increase in distrust leads to less risk reduction.

We show moreover that if all individuals are exposed to the same risk, the political decision and the socially optimal one are different in general but both lie between the individually preferred risk reduction levels.

This last result is not true for differentiated risk exposure. Political decisions in terms of risk reduction may then be extreme and far from optimal if individuals do not trust in public information. Prohibition can then be politically decided even if nobody prefers it, which corresponds to a radical interpretation of the precautionary principle.

To conclude, we think that our model can be considered as a first step in the introduction of behavioral economics insights in a political economics framework. It can be developed in several ways: the assumption of purely opportunistic parties can be relaxed and several risk reduction technologies can be introduced.

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Appendix

A Proof of Proposition 4

(i) λ^{**} is solution of equation (5). Let $e_j = (0;..;0;1;0;..;0)$, with 1 uniquely at the jth place. Then $\lambda^*(\delta_j, y_j)$ is solution of equation (5) when $(\alpha_1, .., \alpha_k) = e_j$. We set:

$$H(\alpha_1, .., \alpha_k, \lambda) = \sum_{i=1}^{k} \alpha_i n_i \left[-T'(\lambda)\frac{y_i}{Y}u'\left(y_i(1-t(\lambda, Y))\right) + p(1-\delta_i)V\right]$$

We have $H(\alpha_1, .., \alpha_k, \lambda^{**}) = 0$, and $H(e_j, \lambda^*(\delta_j, y_j)) = 0$ for any j.
Let j_1, j_2 defined by $H(e_{j_1}, \lambda^{**}) = \min_j H(e_j, \lambda^{**})$ and $H(e_{j_2}, \lambda^{**}) = \max_j H(e_j, \lambda^{**})$.
We have $H(\alpha_1, .., \alpha_k, \lambda) = \sum_{j=1}^{k} \alpha_j H(e_j, \lambda)$. Since $H(\alpha_1, .., \alpha_k, \lambda^{**}) = 0$, thus $H(e_{j_1}, \lambda^{**}) < 0 < H(e_{j_2}, \lambda^{**})$

We can note that $H(e_{j_1}, \lambda)$ is decreasing in λ (since the second order condition of program (5) is satisfied) and $H(e_{j_1}, \lambda^{**}) < 0 = H(e_{j_1}, \lambda^*(\delta_{j_1}, y_{j_1}))$. Thus, $\lambda^{**} > \lambda^*(\delta_{j_1}, y_{j_1})$. The same reasoning allows to prove that $\lambda^{**} < \lambda^*(\delta_{j_2}, y_{j_2})$.
(ii) For CRRA utility functions, for any $R > 0$, $\lambda^{**} < \lambda^*(\delta_i, y_i) \Leftrightarrow \frac{\partial D'_\lambda}{\partial \alpha_i} > 0$ according to Proposition 3 and its proof, where $\frac{\partial D'_\lambda}{\partial \alpha_i} = n_i \left(\frac{-y_i T'(\lambda^{**})}{Y} u'(y_i[1-\frac{T(\lambda^{**})}{Y}]) + (1-\delta_i)pV\right)$
- For $R = 1$, $\frac{\partial D'_\lambda}{\partial \alpha_i} = n_i(-\frac{T'(\lambda^{**})}{Y - T(\lambda^{**})} + (1-\delta_i)pV) = n_i\left[-(1-\overline{\delta})pV + (1-\delta_i)pV\right]$ from (8) and thus $\frac{\partial D'_\lambda}{\partial \alpha_i} = n_i pV(\overline{\delta} - \delta_i)$ i.e. $\lambda^{**} < \lambda^*(\delta_i, y_i) \Leftrightarrow \overline{\delta} > \delta_i$. Similarly, we have $\lambda^{**} = \lambda^*(\delta_i, y_i) \Leftrightarrow \overline{\delta} = \delta_i$ and thus $\lambda^{**} = \lambda^*(\overline{\delta}, \overline{y})$.
- For $R \neq 1$, $\frac{\partial D'_\lambda}{\partial \alpha_i} = n_i \left[-\frac{T'(\lambda^{**})}{Y}\left(1-\frac{T(\lambda^{**})}{Y}\right)^{1-R} y_i^{1-R} + (1-\delta_i)pV\right]$

$= n_i \left[-y_i^{1-R} pV(1-\overline{\delta})\frac{\sum_{j=1}^{k}\alpha_j n_j}{\sum_{j=1}^{k}\alpha_j n_j y_j^{1-R}} + (1-\delta_i)pV\right]$ from (5).

Then, $\frac{\partial D'_\lambda}{\partial \alpha_i} = n_i pV\left[-\frac{u(y_i)}{\bar{u}(y_1,...,y_k)}(1-\overline{\delta}) + (1-\delta_i)\right]$ and thus $\lambda^{**} < \lambda^*(\delta_i, y_i) \Leftrightarrow \frac{u(y_i)}{\bar{u}(y_1,...,y_k)} < \frac{1-\delta_i}{1-\overline{\delta}}$.
Consequently, for $R \neq 1$, $\lambda^{**} < \lambda^*(\overline{\delta}, \overline{y}) \Leftrightarrow \frac{u(\overline{y})}{\bar{u}(y_1,...,y_k)} < 1$.
Note that, for $R > 1$ this means that $u(\overline{y}) > \bar{u}(y_1, ..., y_k)$ i. e. that $u(\overline{y}) > \frac{\sum_{j=1}^{k}\alpha_j n_j u(y_j)}{\sum_{j=1}^{k}\alpha_j n_j}$. This last inequality is true for any u concave. Then $\lambda^{**} < \lambda^*(\overline{\delta}, \overline{y})$ for $R > 1$.
Moreover, for $R < 1$, $\frac{u(\overline{y})}{\bar{u}(y_1,...,y_k)} < 1 \Leftrightarrow u(\overline{y}) < \frac{\sum_{j=1}^{k}\alpha_j n_j u(y_j)}{\sum_{j=1}^{k}\alpha_j n_j}$ which is never true for u concave. Then $\lambda^{**} > \lambda^*(\overline{\delta}, \overline{y})$ for $R < 1$.
(iii) Assume $y_i = y$, $\forall i$. In this case, the first order condition (7) becomes

$$\frac{y}{Y}T'(\lambda)u'\left(y\left[1-\frac{T(\lambda)}{Y}\right]\right) = pV(1-\overline{\delta})$$

and we obtain the first order condition (3) of the program giving the risk level preferred by an individual of trust level $\overline{\delta}$, which implies $\lambda^{**} = \lambda^*(\overline{\delta}, y)$. ■

B Proof of Proposition 5

The reduction is preferred to authorization iff $W^i_{author}(y) \leq W^i_{red}(y)$, which is equivalent to $u(y) - \varepsilon_i b - Q_i \leq u(y(1-t)) - \varepsilon_i b - (1-\lambda)Q_i$, i.e.

$$W^i_{author}(y) \leq W^i_{red}(y) \Leftrightarrow \lambda Q_i \geq u(y) - u(y(1-t)) \tag{9}$$

Prohibition is preferred to authorization iff $W^i_{author}(y) \leq W^i_{prohib}(y)$, which is equivalent to $u(y) - \varepsilon_i b - Q_i \leq u(y(1-\theta))$, i.e.

$$W^i_{author}(y) \leq W^i_{prohib}(y) \Leftrightarrow Q_i \geq u(y) - u(y(1-\theta)) - \varepsilon_i b \tag{10}$$

Prohibition is preferred to reduction iff $W^i_{red}(y) \leq W^i_{prohib}(y)$, which is equivalent to $u(y(1-t)) - \varepsilon_i b - (1-\lambda)Q_i \leq u(y(1-\theta))$, i.e.

$$W^i_{red}(y) \leq W^i_{prohib}(y) \Leftrightarrow (1-\lambda)Q_i \geq u(y(1-t)) - u(y(1-\theta)) - \varepsilon_i b \tag{11}$$

We can represent these 3 inequalities graphically in the plane of coordinates ε_i, Q_i. Note that:

$$\begin{aligned} W^i_{author}(y) &\leq W^i_{red}(y) \Leftrightarrow Q_i \geq C_3 \\ W^i_{author}(y) &\leq W^i_{prohib}(y) \Leftrightarrow Q_i \geq C_2\left(1 - \frac{\varepsilon_i}{B_2}\right) \\ W^i_{red}(y) &\leq W^i_{prohib}(y) \Leftrightarrow Q_i \geq C_1\left(1 - \frac{\varepsilon_i}{B_1}\right) \end{aligned} \tag{12}$$

where we have

$$\begin{aligned} C_3 &= \frac{u(y) - u(y(1-t))}{\lambda} \\ B_2 &= \frac{u(y) - u(y(1-\theta))}{b} \text{ and } C_2 = u(y) - u(y(1-\theta)) \\ B_1 &= \frac{u(y(1-t)) - u(y(1-\theta))}{b} \text{ and } C_1 = \frac{u(y(1-t)) - u(y(1-\theta))}{1-\lambda} \end{aligned}$$

Those 5 parameters are all positive.

We want now, for each couple (ε_i, Q_i), to determine the preferred decision of an agent with type (ε_i, Q_i). We have $Q_i = p_i V_i (1-\varepsilon_i)$, with $\varepsilon_i \in]0;1[$, thus $(\varepsilon_i, Q_i) \in \Delta = \{(\varepsilon_i, Q_i); Q_i \in]0;+\infty[,\ \varepsilon_i \in]0;1[\}$

The domain Δ can be split in 3 zones: $\Delta = E_1 \cup E_2 \cup E_3$. From the inequalities (9), (10), (11), we can easily see that E_1, E_2 and E_3 are convex sets. Thus (i) is proven. However, the union of two of these zones is not necessarily a convex set.

We note that for ε_i and Q_i near 0, the inequalities (9), (10), and (11) are not satisfied. (ε_i, Q_i) is then in the zone E_1. An agent with loss expectation Q_i low is thus always in favor of authorization, if she trusts the government (i.e. ε_i near 0).

Conversely, if Q_i and ε_i are high, then the agent is in favor of prohibition, because the risk Q_i is high, and the trust is low.

Between these two extreme cases, authorization, reduction and prohibition are possible.
- Case 1: if $C_1 \leq C_3$, then $\frac{u(y)-u(y(1-\theta))}{u(y)-u(y(1-t))} \leq \frac{1}{\lambda}$.
- Case 2: if $C_3 \leq C_1$ then $\frac{1}{\lambda} \leq \frac{u(y)-u(y(1-\theta))}{u(y)-u(y(1-t))}$.

With these results, the proof of Proposition 5 is now obvious if we refer to (12) and to the figures corresponding to the different cases.

C Proof of Proposition 6

(i) We set

$$D_z = \sum_{i=1}^{k} n_i \alpha_i W_z(\varepsilon_i, Q_i)$$

where $z = 1$ refers to authorization, $z = 2$ to reduction and $z = 3$ to prohibition.

The policy z^{**} is adopted iff $D_{z^{**}} = \max(D_1, D_2, D_3))$. We have

$$\begin{aligned} D_1 &= \sum_{i=1}^{k} n_i \alpha_i \left[u(y) - \varepsilon_i b - Q_i\right] \\ &= \sum_{i=1}^{k} n_i \alpha_i u(y) - \sum_{i=1}^{k} n_i \alpha_i \varepsilon_i b - \sum_{i=1}^{k} n_i \alpha_i Q_i \\ &= W_{author}(\overline{\varepsilon}, \overline{Q}) \sum_{i=1}^{k} n_i \alpha_i \end{aligned}$$

setting $\overline{\varepsilon} = \frac{\sum_{i=1}^{k} n_i \alpha_i \varepsilon_i}{\sum_{i=1}^{k} n_i \alpha_i}$ and $\overline{Q} = \frac{\sum_{i=1}^{k} n_i \alpha_i Q_i}{\sum_{i=1}^{k} n_i \alpha_i}$

We can similarly check that

$$D_2 = \sum_{i=1}^{k} n_i \alpha_i W_{red}(\overline{\varepsilon}, \overline{Q}) \quad \text{and} \quad D_3 = \sum_{i=1}^{k} n_i \alpha_i W_{prohib}(\overline{\varepsilon}, \overline{Q})$$

This implies that the political decision corresponds to the preference of an "average" agent, i.e. of characteristics $(\overline{\varepsilon}, \overline{Q})$, where $\overline{\varepsilon}$ and $\overline{Q}$ are the averages of the ε_i and the Q_i, computed with the political weight α_i of each agent.

Authorization is then adopted iff $(\overline{\varepsilon}, \overline{Q}) \in E_1$, reduction of risk if $(\overline{\varepsilon}, \overline{Q}) \in E_2$, and prohibition $(\overline{\varepsilon}, \overline{Q}) \in E_3$. The point $(\overline{\varepsilon}, \overline{Q})$ is in the convex hull of the points (ε_i, Q_i), for $i = 1, ..., k$.

(ii) Since E_1, E_2, E_3 are clearly convex sets, we find that if all the groups have the same preferred policy (i.e. belong to the same E_s), then the policy chosen will be this one.

The union of two E_s is not necessarily a convex set, thus the political decision is less obvious if there is no unanimity. We must distinguish several cases.

(iii) In case 1, we note that E_2 is empty and so $conv\{z^*(\varepsilon_i, Q_i), i = 1, ..., k\}$ is equal to E_1 or E_3 or $E_1 \cup E_3$ which are all convex sets.

(iv) In case 2, E_2 is not empty and $E_1\cup\ E_2$, $E_2 \cup E_3$ and $E_1\cup\ E_3$ are clearly not convex sets and then even if $(\overline{\varepsilon}, \overline{Q}) \in conv\{(\varepsilon_i, Q_i), i = 1, ..., k\}$, we may have $z^*(\overline{\varepsilon}, \overline{Q}) \notin conv\{z^*(\varepsilon_i, Q_i), i = 1, ..., k\}$.