Risk Apportionment and Stochastic Dominance[1]

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Abstract

This paper characterizes higher order risk effects, such as prudence and temperance, via preferences that partially order a set of simple 50-50 lotteries. In particular, consider the random variables $\widetilde{X}_N,\ \widetilde{Y}_N,\ \widetilde{X}_M$ and $\widetilde{Y}_M$, and assume that $\widetilde{X}_i$ dominates $\widetilde{Y}_i$ via i^{th}-order stochastic dominance for $i = M, N$. We show that the 50-50 lottery $[\widetilde{X}_N+\widetilde{Y}_M,\ \widetilde{Y}_N+\widetilde{X}_M]$ dominates the lottery $[\widetilde{X}_N+\widetilde{X}_M,\ \widetilde{Y}_N+\widetilde{Y}_M]$ via $(N+M)^{th}$-order stochastic dominance. A preference ranking over these lotteries is shown to characterize higher orders of risk preference. We apply our results in several examples of decision making under risk.

Keywords: Downside Risk, Precautionary Savings, Prudence, Risk Apportionment, Risk Aversion, Stochastic Dominance, Temperance

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1 Introduction

Choice under uncertainty is often model specific. Much debate in the literature discusses the pros and cons of the various types of valuation methods used in modeling such decisions. The choice is made much easier when we have stochastic dominance by one of the alternatives. In such a case, we will have agreement on the optimal choice by a wide range of valuation methods. For example, suppose a corporation believes its shareholders are all risk averse, defined as an aversion to mean-preserving spreads.[1] Its shareholders would then unanimously favor a decision yielding the random payout variable $\widetilde{X}$ over the alternative set of payoffs $\widetilde{Y}$, whenever $\widetilde{X}$ dominates $\widetilde{Y}$ via second-order stochastic dominance (SSD). This holds true for any model of preferences that preserves a preference for SSD. For example, in an expected-utility framework, this would hold whenever the utility function is increasing and concave.

The link between stochastic dominance and preferences within an expected-utility

[1] See Rothschild and Stiglitz (1970).

framework is particularly well known. If we restrict the utility function u to be differentiable, then N^{th}-order stochastic dominance (NSD) of $\widetilde{X}$ over $\widetilde{Y}$ is equivalent to unanimous preference of $\widetilde{X}$ over $\widetilde{Y}$ by any individual whose utility exhibits certain properties. This equivalence has implications for non-expected utility models as well. For example, suppose that all individuals with expected-utility preferences that satisfy NSD preference unanimously prefer $\widetilde{X}$ to $\widetilde{Y}$, then any non-expected utility preference functional that satisfies NSD preference would also lead to the choice of $\widetilde{X}$ over $\widetilde{Y}$.[2]

This paper examines how concepts such as risk aversion, prudence and temperance, as well as higher-order risk effects, can be characterized by a simple lottery preference based upon stochastic dominance rankings. In particular, consider the independent random variables $\widetilde{X}_N,\ \widetilde{Y}_N,\ \widetilde{X}_M$ and $\widetilde{Y}_M$, and assume that $\widetilde{X}_i$ dominates $\widetilde{Y}_i$ via i^{th}-order stochastic dominance for $i = M, N$. We show that the 50-50 lottery $[\widetilde{X}_N + \widetilde{Y}_M,\ \widetilde{Y}_N + \widetilde{X}_M]$ dominates the lottery $[\widetilde{X}_N + \widetilde{X}_M,\ \widetilde{Y}_N + \widetilde{Y}_M]$ via $(N+M)^{th}$-order stochastic dominance. This ranking has implications for choice problems within particular classes of valuation functionals over distribution functions. This characterization is shown to generalize the concept of risk apportionment, as introduced by Eeckhoudt and Schlesinger (2006). We provide examples of such risk apportionment

[2] A similar point was made by Zilcha and Chew (1990).

problems for distributions over wealth and over profits.

The next two sections present our basic model and main results. The subsequent section applies the main result to some choice problems involving risk apportionment.

2 Stochastic Dominance and N^{th} Degree Risk

We start with a definition of stochastic dominance.[3] Assume that all random variables have bounded supports contained within the interval $[a, b]$. Let F denote the cumulative distribution function for such a random variable. Define $F^{(0)}(x) \equiv F(x)$ and define $F^{(i)}(x) \equiv \int_a^x F^{(i-1)}(t)dt$ for $i \geq 1$.

Definition 1 *We say that the distribution F weakly dominates the distribution G in the sense of N^{th}-order stochastic dominance if*

(i) $F^{(N-1)}(x) \leq G^{(N-1)}(x)$ *for all* $a \leq x \leq b$

(ii) $F^{(i)}(b) \leq G^{(i)}(b)$ *for* $i = 1, ..., N-2$.

We write F NSD G to denote F dominates G via N^{th}-order stochastic dominance. If the random variables $\widetilde{X}$ and $\widetilde{Y}$ have cumulative distribution functions denoted by F and G respectively, we will take the liberty to also say that $\widetilde{X}$ NSD $\widetilde{Y}$. For $N = 1, 2, 3$, we will use the more common notations for first-, second-, and third-order stochastic dominance: FSD, SSD and TSD.

[3] See, for example, Ingersoll (1987) and Jean (1980, 1984).

As a special case of stochastic dominance, Ekern (1980) considers the following:

Definition 2 *The distribution G has more N^{th}-degree risk than the distribution F if*

(i) $F^{(N-1)}(x) \leq G^{(N-1)}(x)$ *for all* $a \leq x \leq b$

(ii) $F^{(i)}(b) = G^{(i)}(b)$ *for* $i = 1, ..., N-2$.

Note that G has more N^{th}-degree risk than F is equivalent to saying that F NSD G and the first $N-1$ moments of F and G are identical.[4]

The following two theorems express the well-known links between stochastic dominance and expected utility, as well as Ekern's extension of this result to increases in N^{th} degree risk.[5] Here we let $u(w)$ denote the individual's utility function. For notational convenience, we use $u^{(n)}(w)$ to denote $\frac{d^n u(w)}{dw^n}$.

Theorem 1 *The following are equivalent:*

(i) F *NSD* G

(ii) $\int_a^b u(t)dF \geq \int_a^b u(t)dG$, *for all functions* u *such that sgn* $u^{(n)}(w) = (-1)^{n+1}$ *for* $n = 1, ..., N$.

[4] The second condition follows easily from part (ii) in the definition by integrating both $F^{(i)}$ and $G^{(i)}$ by parts. The case where $N = 3$ is labeled as an "increase in downside risk" and given special attention in a paper by Menezes et al. (1980). The case where $N = 4$ is examined in part by Menezes and Wang (2005).

[5] See Hadar and Russell (1969) and Hanoch and Levy (1969) who introduced this notion into the economics literature for SSD. See Jean (1980) and Whitmore (1989), as well as Ingersoll (1987) for extensions to higher orders of stochastic dominance.

Theorem 2 *The following are equivalent:*

(i) G has more N^{th} degree risk than F

(ii) $\int_a^b u(t)dF \geq \int_a^b u(t)dG$, *for all functions u such that sgn* $u^{(N)}(w) = (-1)^{N+1}$.

3 Main Result

In this section, we examine a particular lottery preference over random wealth variables that can be ordered via stochastic dominance. We show how this lottery preference relates to higher order risk effects, such as prudence and temperance. Our characterization is a generalization of the concept "risk apportionment," as introduced by Eeckhoudt and Schlesinger (2006). In particular, their characterization is shown to be a special case of our results here.

Eeckhoudt and Schlesinger (2006) consider two basic "harms," namely a sure loss and a zero-mean risk. They start from an arbitrary (possibly random) initial wealth level and consider two alternative 50-50 lotteries that will be added to the initial wealth: the first consisting of a 50-50 chance of either one harm or the other harm, and the second consisting of a 50-50 chance of adding either both harms simultaneously or adding neither. An individual is defined as being "prudent" if he or she always would prefer the first of these two lotteries, which they label as a desire to "disaggregate" these two harms. They then show how this definition, which they also label as "risk apportionment of order 3," is equivalent to the existing definition of prudence as given by Kimball (1990).

Replacing the sure loss above with a second zero-mean risk that is statistically independent from the first, they then show that a desire to disaggregate these two harms (i.e., preference for a 50-50 lottery adding one risk or the other risk over a 50-50 lottery adding the sum of both risks or adding neither) is equivalent to the definition of "temperance" as described by Kimball (1992) and Gollier and Pratt (1996). This lottery preference is also labeled as "risk apportionment of order 4."

Proceeding recursively and allowing the 50-50 lotteries generated to be considered the relative "harms," Eeckhoudt and Schlesinger characterize "risk apportionment of order N." Moreover, it is shown that preferences satisfy risk apportionment of orders $1, ..., N$ if and only if they also satisfy a preference for N^{th}-order stochastic dominance. In what follows, we essentially generalize the Eeckhoudt and Schlesinger result to allow for the relative harms to be any random variables that can be ranked via some order of stochastic dominance.

Let $[A, B]$ denote a lottery that pays either A or B, each with probability one-half. Consider the mutually independent random variables $\widetilde{X}_N$, $\widetilde{Y}_N$, $\widetilde{X}_M$ and $\widetilde{Y}_M$, and assume that $\widetilde{X}_i$ dominates $\widetilde{Y}_i$ via i^{th}-order stochastic dominance for $i = M, N$. We wish to compare the 50-50 lotteries $[\widetilde{X}_N + \widetilde{Y}_M,\ \widetilde{Y}_N + \widetilde{X}_M]$ and $[\widetilde{X}_N + \widetilde{X}_M,\ \widetilde{Y}_N + \widetilde{Y}_M]$.

Theorem 3 *Suppose that $\widetilde{X}_i$ dominates $\widetilde{Y}_i$ via i^{th}-order stochastic dominance for $i = M, N$. The lottery $[\widetilde{X}_N + \widetilde{Y}_M,\ \widetilde{Y}_N + \widetilde{X}_M]$ dominates the lottery $[\widetilde{X}_N + \widetilde{X}_M,\ \widetilde{Y}_N + \widetilde{Y}_M]$ via $(N+M)^{th}$-order stochastic dominance.*

Proof. Let T be a positive integer and define $U_T \equiv \{u \mid sgn\ u^{(n)}(w) = (-1)^{n+1}$ for $n = 1, ..., T\}$. For an arbitrary function $u \in U_{N+M}$ define $v(w) \equiv Eu(\widetilde{Y}_M + w) - Eu(\widetilde{X}_M + w)$, where E denotes the expectation operator. We first show that $v \in U_N$. To see this, consider any integer k, $1 \leq k \leq N$. Observe that $u^{(k)} \in U_{N+M-k} \subset U_M$. Now $sgn\ v^{(k)}(w) = sgn\ [Eu^{(k)}(\widetilde{Y}_M + w) - Eu^{(k)}(\widetilde{X}_M + w)] = (-1)^{k+1}$. The second equality above follows since $u^{(k)} \in U_M$ and $\widetilde{X}_M$ MSD $\widetilde{Y}_M$. Thus, $v \in U_N$.

The condition that $\widetilde{X}_N$ dominates $\widetilde{Y}_N$ via N^{th}-order stochastic dominance, together with $v \in U_N$, implies that $Ev(\widetilde{X}_N) \geq Ev(\widetilde{Y}_N)$, which by the definition of v is equivalent to

$$Eu(\widetilde{X}_N + \widetilde{Y}_M) - Eu(\widetilde{X}_N + \widetilde{X}_M) \geq Eu(\widetilde{Y}_N + \widetilde{Y}_M) - Eu(\widetilde{Y}_N + \widetilde{X}_M). \tag{1}$$

Rearranging terms above, this inequality is equivalent to

$$\frac{1}{2}\{Eu(\widetilde{X}_N + \widetilde{Y}_M) + Eu(\widetilde{Y}_N + \widetilde{X}_M)\} \geq \frac{1}{2}\{Eu(\widetilde{X}_N + \widetilde{X}_M) + Eu(\widetilde{Y}_N + \widetilde{Y}_M)\}, \tag{2}$$

which is precisely the lottery preference claimed in the theorem. ■

The lottery preference expressed in Theorem 3 is analogous to the notion of "disaggregating the harms" discussed by Eeckhoudt and Schlesinger (2006), if we interpret the "harms" as sequentially replacing each of the $\widetilde{X}$ random variables with a $\widetilde{Y}$ random variable in the sum $\widetilde{X}_N + \widetilde{X}_M$. Or, said differently, it expresses a preference for lotteries that combine relatively good assets with relatively bad ones.

The preferences described here lead to a partial ordering of the four alternative sums of random variables, based upon stochastic dominance criteria:

$$\widetilde{X}_N + \widetilde{X}_M \succ \widetilde{X}_i + \widetilde{Y}_j \succ \widetilde{Y}_N + \widetilde{Y}_M \quad \text{for } (i,j) \in \{(M,N),(N,M)\}. \tag{3}$$

Note that the sums $\widetilde{X}_M + \widetilde{Y}_N$ and $\widetilde{X}_N + \widetilde{Y}_M$ cannot be ordered via stochastic dominance. In the spirit of Menezes and Wang (2005) we can refer to these two sums as the "inner risks" and the sums $\widetilde{X}_N + \widetilde{X}_M$ and $\widetilde{Y}_N + \widetilde{Y}_M$ as the "outer risks." Theorem 3 thus expresses a preference for a 50-50 lottery over the two inner risks as opposed to a 50-50 lottery over the two outer risks.[6]

The following Corollary extends this result to Ekern's ordering by N^{th}-degree risk. The proof follows easily from the proof of Theorem 3.

Corollary 4 *Suppose that $\widetilde{Y}_i$ has more i^{th}-degree risk than $\widetilde{X}_i$ for $i = M, N$. Then*

[6] A similar analogy is made by Eeckhoudt et al. (2007), who describe a lattice structure on the ranking of the lottery components and subsequently define a preference functional over the lattice as being submodular.

the lottery $[\widetilde{X}_N + \widetilde{X}_M,\ \widetilde{Y}_N + \widetilde{Y}_M]$ *has more* $(N+M)^{th}$*-degree risk than the lottery* $[\widetilde{X}_N + \widetilde{Y}_M,\ \widetilde{Y}_N + \widetilde{X}_M]$.

4 Applications

In this section we illustrate the applicability of our results. In particular, we first demonstrate how Theorem 3 and its corollary can be used to gain insight into a few extant concepts such as downside risk aversion and other higher-order effects of risk preferences. We then turn to two economic examples that directly apply our results to decision making under risk.

4.1 Aversion to Downside Risk

We first use an illustration that is the result of an experiment by Mao (1970), and was used by Menezes et al. (1980) to motivate the concept of aversion to downside risk (i.e. prudence). Consider the following two lotteries. Lottery A pays 1000 with a probability of $\frac{3}{4}$ and pays 3000 with a probability of $\frac{1}{4}$. Lottery B pays zero with a probability of $\frac{1}{4}$ and pays 2000 with a probability of $\frac{3}{4}$. Note that both lotteries exhibit the same first two moments in their distributions. Individuals who prefer lottery A to lottery B exhibit "downside risk aversion."[7]

This lottery preference follows from Corollary 4 by defining

$$\widetilde{X}_N \equiv 2000$$

$$\widetilde{Y}_N \equiv 1000$$

$$\widetilde{X}_M \equiv 0$$

$$\widetilde{Y}_M \equiv [-1000, +1000], \text{ a 50-50 lottery.}$$

Clearly $\widetilde{Y}_N$ is a first-order increase in risk over $\widetilde{X}_N$ and $\widetilde{Y}_M$ is a second-order increase in risk over $\widetilde{X}_M$. It follows that A is the 50-50 lottery $[\widetilde{X}_N + \widetilde{Y}_M,\ \widetilde{Y}_N + \widetilde{X}_M]$ and B is the 50-50 lottery $[\widetilde{X}_N + \widetilde{X}_M,\ \widetilde{Y}_N + \widetilde{Y}_M]$. Thus, from our Corollary, lottery B displays more third-order risk, i.e. displays more downside risk, so that anyone who is prudent (with $u''' > 0$ in an expected utility framework) would prefer lottery A.

4.2 Higher-order Effects of Risk

Within expected-utility models, the signs of the derivatives on the utility function all have some economic meaning. Temperance, $u^{(4)} < 0$, is a fourth-order effect.

[7] A recent experimental paper by Baltussen et al. (2006) also supports this approach, though unknowingly. Although they interpret their experiments for Prospect Theory, they use mean preserving spreads for "gains" and "losses," which we can reinterpret as "low wealth" and "high wealth" in our setting. We can interpret their results as showing a preference for attaching a mean-preserving spread to the higher wealth level, which is the equivalent of aversion to downside risk in our setting.

Theorem 3 can be used to give two alternative equivalences to temperance.

Let $X_a = X_b = 0$ and let $\widetilde{Y}_a$ and $\widetilde{Y}_b$ be independent zero-mean risks. Thus X_i dominates $\widetilde{Y}_i$ by SSD for $i = a, b$. Temperance implies that the 50-50 lottery $[X_a + \widetilde{Y}_b, X_b + \widetilde{Y}_a]$ is preferred to the lottery $[X_a + X_b, \widetilde{Y}_a + \widetilde{Y}_b]$. Indeed, this lottery preference *defined* "temperance" in Eeckhoudt and Schlesinger (2006). The individual prefers a 50-50 gamble between $\widetilde{Y}_a$ and $\widetilde{Y}_b$ over a 50-50 gamble between $\widetilde{Y}_a + \widetilde{Y}_b$ or neither. By our Theorem 3, we know that this lottery preference is equivalent to temperance *for any* $\{\widetilde{X}_a, \widetilde{X}_b, \widetilde{Y}_b, \widetilde{Y}_a\}$ with $\widetilde{X}_i$ dominating $\widetilde{Y}_i$ by SSD for $i = a, b$, not just the particular set $\{\widetilde{X}_a, \widetilde{X}_b, \widetilde{Y}_b, \widetilde{Y}_a\}$ as described above.

We can obtain a second equivalence for temperance by letting $\widetilde{Y}_a$ denote an increase in downside risk over $\widetilde{X}_a$ (see Section 4.1 above), and letting $\widetilde{X}_b$ FSD $\widetilde{Y}_b$. The (stochastically) higher wealth in $\widetilde{X}_b$ helps to "temper" the effects of the increased downside risk in $\widetilde{Y}_a$. Thus, we prefer to pair $\widetilde{X}_b$ together with $\widetilde{Y}_a$ in our lottery preference.

Lajeri (2004) studies the effects of background risks on precautionary savings and in doing so, examines the condition of decreasing absolute temperance. A necessary condition for this property is $u^{(5)} > 0$, which she labels as "edginess." By choosing $M = 1$ and $N = 4$, we can use Theorem 3 to interpret edginess as implying that a (stochastic) increase in wealth helps to temper the effects of an increase in fourth-order risk. By choosing $M = 2$ and $N = 3$, we can alternatively interpret edginess as implying that a decrease in risk (via SSD) helps to temper the effects of an increase in downside risk.

4.3 Precautionary saving

Consider a simple two-period model of consumption and saving. An individual has a random income of $\widetilde{X}$ at date $t = 0$ and income $\widetilde{Y}$ at date $t = 1$. The individual decides to save some of her income at date $t = 0$ and to consume the rest. She must decide on how much to save before learning the realized value of $\widetilde{X}$. Thus, her consumption at date $t = 0$ is $\widetilde{X} - s$, where s is the amount saved. If $s < 0$, the consumer is borrowing money (i.e. negative savings) and consuming more than the realized value of $\widetilde{X}$ at date $t = 0$. We assume that the interest rate for borrowing or lending is zero and that there is no time-discounting for valuing consumption at date $t = 1$. At this date the individual consumes her income plus any savings, $\widetilde{Y} + s$. Let s^* denote the individual's optimal choice for savings.

Suppose that $\widetilde{X}$ dominates $\widetilde{Y}$ via N^{th}-order stochastic dominance. For any nonnegative scalar $\varphi \geq 0$, since φ dominates $-\varphi$ by FSD, it follows from Theorem 3 that the 50-50 lottery $[(\widetilde{X} - \varphi), (\widetilde{Y} + \varphi)]$ dominates $[(\widetilde{X} + \varphi), (\widetilde{Y} - \varphi)]$ by (N+1)SD. Reinterpreting the "50-50 lottery" $[A, B]$ as sequential consumption of A at $t = 0$

and B at $t = 1$, Theorem 3 implies that saving an arbitrary amount $\varphi \geq 0$ always dominates saving the amount $-\varphi$, whenever preferences satisfy $(N+1)^{th}$-degree stochastic dominance preference. It follows that we must have $s^* \geq 0$ for this individual.

For example, suppose that one's income is risky in both periods, but it is riskier in the sense of SSD next period. Anyone with preferences satisfying third-order stochastic dominance preference would prefer to save money rather than to borrow money.[8] If instead of SSD we assume that next period's income is stochastically higher than this period's via FSD, then any individual who is risk averse (i.e. satisfying SSD preference) will prefer to borrow rather than save.

4.4 After-tax profits

Consider a risk neutral corporation with taxable profit $\widetilde{X}$ in country A and taxable profit $\widetilde{Y}$ in country B. We assume that the tax schedule is identical in both countries and that $\widetilde{X}$ SSD $\widetilde{Y}$. The tax owed on realized profit π is denoted by $t(\pi)$. The tax schedule is assumed to be increasing with a marginal tax rate that is also increasing, but at a decreasing rate.[9] After tax profits can thus be written as $u(\pi) = \pi - t(\pi)$.

[8]For example, suppose that $\widetilde{X}$ is a constant equal to $E\widetilde{Y}$ and that preferences are given by expected utility. Then this result coincides with that of Leland (1968) and Sandmo (1970), for the case where preferences display prudence, $u''' > 0$.

[9]This assumption is realistic since the marginal rate is often bounded by some maximum, such as fifty percent of additional profit.

If $t(\cdot)$ is differentiable, our assumptions about t imply that $u''(\pi) < 0$ and $u'''(\pi) > 0$. Moreover, since we should also have $t'(\pi) < 1$ for any profit level π, it follows that $u'(\pi) > 0$ as well.

Suppose now that the corporation has a new project with a pre-tax distribution of profit $\widetilde{Z}$, where $\widetilde{Z} > 0$ a.s. The corporation must decide whether to locate the project in country A or in country B. The after-tax total profit of the corporation is given by $u(\pi_A) + u(\pi_B)$, where π_i denotes the realized pre-tax profit in country i, for $i = A, B$. Since $\widetilde{Z}$ dominates zero by FSD, it follows from Theorem 3 that $E[u(\widetilde{X}) + u(\widetilde{Y} + \widetilde{Z})] > E[u(\widetilde{X} + \widetilde{Z}) + u(\widetilde{Y})]$, since the valuation function u (i.e. after-tax profits) satisfies third-order stochastic dominance preference. Thus, the firm should locate the new project in country B in order to maximize its global after-tax profit.

5 Concluding Remarks

Hedging has long been a capstone of risk-management strategy. Faced with an initially risky wealth prospect, one can add a position in another asset whose payoff is negatively correlated to the original risky wealth. Such an asset will usually have a positive net payoff in states of the world in which the random wealth would otherwise have been low. If we assume risk aversion, then marginal utility in low-wealth states is higher and therefore such a hedge is valuable to a risk averter. The cost of such hedging is that the net payoff is usually negative in states of the world in which the random wealth would otherwise have been high. In a certain sense, hedging pairs up relatively good outcomes on one asset with relatively bad outcomes on another asset, and vice versa.

By contrast, risk apportionment pairs relatively good assets with relatively bad ones. It assumes that the payoffs on the assets are independent, which is antithetical to the premise of hedging. Given the partial ordering in (3), we prefer a 50-50 gamble between the two "inner risks" as opposed to the two outer ones. Here relatively "good" and "bad" are determined via stochastic dominance rankings. Under expected utility, this ranking coincides with a utility function (or some other valuation function) whose derivatives alternate in sign. If these rankings hold for all orders of stochastic dominance, then they would coincide with preferences that are completely monotone, as described by Brockett and Golden (1987) and Pratt and Zeckhauser (1987).

We provided several interpretations and applications of our main theoretical result, Theorem 3. Our results also can be used to add intuition to many other extant concepts in the literature, such as skewness preference (see Chiu 2005) and transfer principles in income redistribution (see Fishburn and Willig 1984 and Moyes 1999).

Moreover, our results can be useful in situations where preferences over the lotteries are reversed. For example, a decision maker might prefer mean-preserving increases in risk over the domain of losses, or might prefer increases in downside risk over the domain of gains.[10] The equivalences in our paper easily allow for such adaptation.

The lottery preference described in this paper generalizes the concept of "risk apportionment," as introduced by Eeckhoudt and Schlesinger (2006). One strength of their result is the simplicity of their characterization of lower levels of risk apportionment via a preference over simple 50-50 lotteries. However, their characterization is less simple for higher orders, since it requires that they construct larger and larger nestings of simple lotteries. A strength of this paper is that we can characterize any degree of risk apportionment by a preference between two simple 50-50 lotteries.

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[10] For example, Wong (2007) essentially defines the first three orders of stochastic dominance over the de-cumulative distribution function and refers to these respectively as "descending stochastic dominance" of each order.

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